ASTE 101 Fall 2008
Homework 6 (60 points)
Due in class Wednesday 22 October 2008

1. (20 points)

   (a) The Global Positioning System (GPS) satellites are in circular orbits with inclination 55 degrees and altitude 20200 km. One particular GPS satellite has right ascension of ascending node $\Omega = 165.7°$. The satellite passes the ascending node at $t = 0$, and this happens to be at a time when the vernal equinox vector lines up with zero longitude on Earth (for example, it might be noon on March 21 in Greenwich, England), so that Ω and the spacecraft's longitude in Earth's coordinates are the same at $t = 0$. What are the longitude and latitude of the satellite's nadir (point on Earth just below satellite) at time $t = 13850$ s? You can most easily do this in Matlab using my function `vectorAtTrueAnomaly` (a file in `ViewEarth.zip`). (Note that since this is a circular orbit, there is no perigee, so we consider that the argument of perigee, ω, is zero, and we measure the true anomaly from the ascending node.) Here are a few useful points:

   - The function `vectorAtTrueAnomaly` takes arguments in radians.
   - For a circular orbit, the true anomaly at time t is $2\pi t/P$.
   - The function `vectorAtTrueAnomaly` returns a vector (array of length 3, (x, y, z)) in coordinates such that x is the distance along the vernal equinox vector, z is the distance north, and y is the distance perpendicular to both. On a map of Earth, with longitude 0 and latitude 0 in the middle, y is to the right (east), z is up (north), and x points out of the map.
   - Given the coordinates (x, y, z), the longitude is given by the Matlab function `atan2(y,x)`, and the latitude is given by $sin^{-1}(z/\sqrt{x^2 + y^2 + z^2})$.
   - There is a correction because of Earth's rotation. From the longitude just given, you need to subtract $2\pi t/D_{\text{sid}}$ (where $D_{\text{sid}} = 86164$ s is a sidereal day) because Earth has rotated that much to the east in time t.

   (b) Write a loop in Matlab to compute the longitude and latitude of this spacecraft every 100 s over three orbits, storing them in arrays. Convert them to degrees and plot them on top of a map of Earth using `drawEarthMap` (posted on website). (Note that when you apply the Earth rotation correction, you may end up with values for the longitude that are less than -180 degrees. When this happens you must add 360 degrees to put it in the range (-180,180).)
   Extra credit Your plot has a spurious line on it because of the jump from 180 to -180 degrees. Figure out how to notice this jump and not draw it.

2. (20 points) A replacement GPS satellite is launched and placed in a parking orbit at altitude 150 km and inclination 28.5 degrees. It is to be lifted to the GPS orbit using a Hohmann transfer with combined plane change at apogee of the transfer orbit. The perigee and apogee of the transfer orbit are to be in the equatorial plane.

(a) What is the speed of the parking orbit?

(b) What is the speed of the GPS orbit?

(c) What are the perigee and apogee speeds of the Hohmann transfer orbit?

(d) What are the two speedups ΔV_1 and ΔV_2? Remember that the second ΔV must be calculated to achieve the plane change (rotation by the difference of inclinations) as well as speedup to achieve the final circular GPS orbit.

3. (20 points) Set up a scenario in STK to simulate the previous problem of lifting a S/C to GPS, including parking orbit, transfer orbit, and final GPS orbit. Turn in a plot of the ground track of the entire scenario over three parking orbits, transfer orbit, and three GPS orbits. Note that this is similar to the scenario I showed in class that simulates the LEO-GEO transfer.